Lattice Formulations of Chen-Simons Theory

Topologically ordered systems can be described by discrete gauge theories, such a $\mathbb{Z}_2$ gauge theory, or continuous gauge theories will a Chern-Simons (CS) term. Additionally, a gauge field with a CS term coupled to matter can produce quasiparticles with fractional statistics and charge. This suggests a lattice formulation of pure CS theory might be useful when trying to describe topologically ordered phases in lattice systems. While $\mathbb{Z}_2$ gauge theory is more naturally formulated on a lattice, the CS term does not easily translate to a lattice. This can be seen from the fact that the CS term couples charges to fluxes; the charged matter fields live on the sites of the lattice, where as the flux of the gauge field lives on the plaquettes. As each charge borders on several plaquettes, one must decide how to couple the charge to flux in a consistent way. The most direct lattice discretizations of pure CS theory have either have problematic divergences or are not gauge invariant. These problems can be corrected, but this requires adding a Maxwell term, making the CS term non-local, or doubling the gauge field to restore parity invariance. Additionally, a compact lattice version of CS theory has only been formulated in the dual theory [Diamantini].

The two most direct lattice discretizations of (non-compact) Chern-Simons theory were proposed by Fradkin, and Frohlich, Marchetti and Luscher. Fradkin's lattice CS term is not gauge invariant [Fradkin], even on a system without a boundary. Frohlich and Marchetti were able to write down a gauge invariant lattice CS term [Frohlich and Marchetti]. This was formulated into a full lattice version of Maxwell CS theory by Luscher [Luscher]. However, when the Maxwell term vanishes, by taking the electric charge to infinity, divergences appear in the the commutation relations of the gauge fields and the current-current correlator. These divergences are related to extra zeros that exist in the kernel of the CS term.

To correct these problems arising in pure CS theory, the CS term must be made non-local or parity invariant. Eliezer and Semenoff were able to correct the problems that previously arose in pure CS theory of the lattice [Eliezer] but this requires a non-local CS term. Kantor and Susskind formulated a local pure CS theory by two gauge fields of opposite parity[Kantor]. In fact Berruto, Diamantini, and Sodano proved that any local and gauge invariant pure CS theory with odd parity will have extra zeros in the kernel that make the action non-integrable [Berruto ].

Consequently, recent work on lattice models with topological order has focused on models formulated in terms of the gauge invariant Wilson loops, where the underlying CS gauge theory is implicit. There also have been lattice formulations of CS theory with discrete (both Abelian and non-Abelian) gauge groups [Doucot].