Quantum Spin Hall effect

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1) The Intrinsic Spin Hall effect in 3D. Using semiclassical equations of motion following Sinova we talk about the Intrinsic spin hall effect.

Since in the presence of Rashba spin orbit coupling spin is not a conserved quantity it is difficult to characterize spin conductivity using the Kubo formula. So following Zhang and Murakami we define the conserved spin current and use the Kubo formula to characterize the spin conductance.

2) The Spin Hall effect in 2D in Graphene: Next we use the 2D Dirac equation model for graphene and notice that similar to the case of the 3D spin hall effect one can calculate a spin hall conductance using the kubo formula and show that it is Quantized in exactly the same way as the Integer Quantum Hall effect in the absence of Rashba coupling.

3) Haldane model: Next we look at the Haldane model numerically and show that it has edge states in a way similar to the Quantum Hall case. We re-examine Laughlin’s argument to show that these edge states lead to a spin hall coefficient whose value is determined by the property of the edge states.

4) $Z_2$ topological characterization: The charge Quantum Hall effect could be understood as arising due to topologically invariant chern numbers associated with the response of the state ground state to twisted boundary conditions. In particular the integer quantum hall effect could be understood as a Chern number characterization of the states in the Brillouin Zone. We follow Kane and Mele’s argument to show that in non-interacting systems with time reversal symmetry there is a topologically invariant integer defined modulo 2 which distinguishes conventional insulators from the Spin hall state graphene is in.

5) Chern number classification: Recently S C Zhang obtained a generalization of the above topological argument which argues that bulk topological order defined by the first chern number is protected by the bulk gap and always manifests itself through the existance of gapless edge states. However he finds that the gaplessness of the edge states in open boundary systems requires conditions in addition to bulk characteristics. We examine his argument in the context of the conventional charge integer quantum hall and how it is connected to the $Z_2$ topological characterization for time reversal invariant systems.