Two-Dimensional Anisotropic Non-Fermi-Liquid Phase of Coupled Luttinger Liquids

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(Received 14 March 2000)

Using bosonization techniques, we show that strong forward scattering interactions between one-dimensional spinless Luttinger liquids (LL) can stabilize a phase where charge-density wave, superconducting, and transverse single particle hopping perturbations are irrelevant. This new phase retains its LL-like properties in the directions of the chains, but with relations between exponents modified by the transverse interactions, whereas it is a perfect insulator in the transverse direction. The mechanism that stabilizes this phase is strong transverse charge-density wave fluctuations at incommensurate wave vector, which frustrates crystal formation by preventing lock-in of the in-chain density waves.

DOI: 10.1103/PhysRevLett.86.676
PACS numbers: 71.10.Pm, 71.10.Hf, 71.27.+a

Interacting fermions in one dimension can exhibit Luttinger liquid behavior where, in contrast to Fermi liquids, all the low lying excitations are collective modes [1]. The search for such non-Fermi-liquid behavior in higher dimensions prompted several authors to study the problem of coupled Luttinger liquids. But renormalization group (RG) studies such as [2] found the coupling to destabilize the Luttinger liquid behavior. It was, however, argued by Strong et al. that, in spite of its relevance in the RG sense, electron transverse hopping may be incoherent [3], allowing for a non-Fermi liquid in dimensions greater than one. In addition to these theoretical motivations, interacting Luttinger liquids could well arise in experimental systems like quasi-one-dimensional organic conductors and in “ropes” of nanotubes. Recently, coupled one-dimensional systems have also emerged in the stripe phases of quantum Hall systems in higher Landau levels [4] and in the cuprates [5].

In this paper, we revisit the problem of coupled Luttinger liquids and address the question of the existence of a stable two-dimensional phase that retains some of the properties of the one-dimensional Luttinger liquid. We consider this issue in detail within the RG framework using bosonization for the case of spinless fermions. Although most of the physical systems of interest are of spinful (rather than spin polarized or spinless) fermions, that case is techni- cally more involved due to the presence of exchange interactions between the chains, and we postpone its study to [6]. The case of an infinite set of coupled spinless Luttinger liquids is simpler but should still capture some of the general physics present in the spinful case. There is also a possibility that such systems may be directly realized by spin polarizing a layered quasi-one-dimensional system of low electron density with an in-plane magnetic field or in the spin polarized Luttinger liquid formed by Zeeman split quasiparticles bound to the superconductor vortex core [7,8]. The new ingredient in our study is the inclusion of strong forward scattering interactions between the chains. We then find that an anisotropic phase, which is Luttinger liquid like along the chains but a perfect charge insulator in the transverse direction, is stable against a range of instabilities, including those that could lead to a superconductor (SC), crystal, or 2D Fermi liquid phase. Hence this new phase is a non-Fermi liquid phase in two dimensions that is highly anisotropic. In the direction of the chains the correlation functions have power law forms with nontrivial exponents, as in a Luttinger liquid. However, the relations between exponents are modified, as compared with completely decoupled Luttinger liquids, by the strong forward scattering interactions between the chains. We call this phase the sliding Luttinger liquid (SLL). It is analogous to the sliding phase found by O’Hern et al. [9] in the related problem of XY models coupled by suitable gradient interactions, which motivated our approach. In contrast to [10], we consider transverse hopping operators when establishing the stability of the SLL.

The stable sliding Luttinger liquid fixed points that we find occur close to an instability towards transverse charge-density wave (CDW) ordering. The instability occurs when the stiffness of the density fluctuation mode at a particular transverse wave vector vanishes. We propose a physical mechanism that is responsible for stabilizing this phase—i.e., that the proximity to the transverse charge-density wave state induces strong fluctuations of the local in-chain density, which frustrate crystal formation. This mechanism is most effective if the wavelength of the transverse charge-density wave phase is incommensurate with the spacing between the chains.

The existence of this phase may at first sight contradict previous results on coupled Luttinger liquids. However, previous approaches focused on either the simpler case of two interacting chains [2] or the perturbative effect of these transverse interactions. Indeed, as we show this phase can exist when strong forward scattering couplings between nearest and (at least) second nearest neighbor chains are taken into account.

The sliding Luttinger liquid.—We consider an anisotropic 2D system composed of parallel chains (labeled by an integer \( i \)) with spinless Luttinger liquids in each chain. The bosonized form of the fermion operators...
near the right and left Fermi points are
\[ \psi_{R/L,i}(x) = \frac{1}{\sqrt{4\pi\epsilon}} e^{i\phi_{i}(x)} \chi_{R/L,i}, \]
where \( \epsilon \) is some intrachain cutoff, the \( \chi_{R/L,i} \) are the Klein factors, and \( x \) is the coordinate along the chain. The Luttinger liquid on each chain is described by the usual Lagrangian:
\[ L_0^{(i)} = \frac{K_0}{2} \left( [\partial_x \Theta_i(x,t)]^2 - [\partial_x \Theta_i(x,t)]^2 \right), \]
where \( \Theta_i = (\phi_{R,i} - \phi_{L,i})/\sqrt{4\pi} \) and \( K_0 \) is an interaction dependent constant with \( K_0 > 1 \) for repulsive intrachain interactions. The sound velocity on each chain has been set to unity. In the following, we also use the dual field \( \Phi_i(x,t) = (\phi_{R,i} + \phi_{L,i})/\sqrt{4\pi} \).

We now add forward scattering interactions between the chains which correspond to couplings between the long wavelength components of the densities \( \rho_i(x,t) \) and of the currents \( J_i(x,t) \):
\[ L_{\text{int}} = 2\pi \sum_{i \neq j} \left[ J_i K_{ij} J_j - \rho_i K_{ij}^0 \rho_j \right], \]
where the interactions are assumed local in \( x \) and time [11]. Using the bosonization relations \( \sqrt{4\pi} \rho_i(x,t) = \partial_x \Theta_i(x,t) \) and \( \sqrt{4\pi} J_i(x,t) = -\partial_x \Theta_i(x,t) \), we obtain the bosonized form of the Lagrangian \( L_{\text{tot}} = \sum_i L_i^{(i)} + L_{\text{int}} \) for the interacting Luttinger liquids:
\[ L_{\text{tot}} = \frac{1}{2} \sum_{i,j} \left[ (\partial_x \Theta_i) K_{ij}^0 (\partial_x \Theta_j) - (\partial_x \Theta_i) K_{ij}^0 (\partial_x \Theta_j) \right], \]
where the coupling matrices are defined by \( K_{ij}^0 = K_0 \delta_{ij} + K_{ij}^0 \). By introducing Fourier transforms in the direction transverse to the chains, this Lagrangian can be rewritten as
\[ L_{\text{tot}} = \int_{q_\perp} \frac{K(q_\perp)}{2} \left[ \frac{1}{v(q_\perp)} [\partial_x \Theta_i q_\perp]^2 - [\partial_x \Theta_i q_\perp]^2 \right] \]
with the notation \( v(q_\perp) = \int \frac{dq_\perp}{2\pi} \) (the transverse chain spacing has been set to unity). The stiffness \( K(q_\perp) \) is defined by \( K(q_\perp) = \sqrt{K^T(q_\perp) K^T(q_\perp)} \) and the velocity \( v(q_\perp) = \sqrt{K^T(q_\perp) K^T(q_\perp)} \). Note that Lorentz invariance corresponds to \( K^T(q_\perp) \propto K^T(q_\perp) \) (as in the isotropic model studied in [9]). In that case, all modes have the same velocity.

The Lagrangian (3) is invariant under the transformations \( \Phi_i \to \Phi_i + c_i \) and \( \Theta_i \to \Theta_i + d_i \), where \( c_i \) and \( d_i \) are constants on each chain. We thus call the corresponding fixed point a SLL fixed point [12]. From this symmetry we deduce that the total numbers of left (right) moving fermions on each chain are good quantum numbers and expectation values of operators that change these—as such as \( \langle \psi_{L,i} \psi_{L,j} \rangle \) for \( i \neq j \)—are necessarily zero in this phase. This corresponds to a perfect charge insulator in the transverse direction. Density (and current) correlations in the transverse direction are, however, nontrivial. For short ranged density and current interactions, they decay exponentially with separation between the chains. The low energy modes are density oscillations (sound) with dispersion \( E(q_\parallel, q_\perp) = v(q_\perp) |q_\parallel| \) (where \( q_\parallel \) is the wave vector along the chain) which in general will propagate both parallel and perpendicular to the chains. These modes can, for instance, transport heat perpendicular to the chains although the system is a perfect charge insulator in that direction. Correlation functions along the chains exhibit power law behavior as in the Luttinger liquid. However, the exponents now depend on the function \( K(q_\perp) \) rather than on a single number as in the case of completely decoupled Luttinger liquids (1).

Therefore, relations between exponents that are valid for decoupled Luttinger liquids no longer hold in this case.

As this phase is described by a Gaussian Lagrangian, we can study perturbatively the possible relevance of various operators to ascertain its stability.

**Transverse hopping operators.**—In the usual case of two chains, it is easily shown that the most relevant operators correspond either to single particle (SP), particle-hole (CDW), or pair hopping (SC) [2]. It is thus natural to first focus on these operators in our stability analysis. They are defined, respectively, by
\[ \delta L_{\text{sp}} = \sum_{i,j} t_{ij} \left( \psi_{R,i} \psi_{R,j} + \psi_{L,i} \psi_{L,j} + \text{H.c.} \right), \]
\[ \delta L_{\text{cdw}} = \sum_{i,j} g_{ij} \left( \psi_{R,i} \psi_{L,j} + \psi_{L,i} \psi_{R,j} + \text{H.c.} \right), \]
\[ \delta L_{\text{sc}} = \sum_{i,j} g_{ij} \left( \psi_{L,i} \psi_{R,j} + \psi_{R,i} \psi_{L,j} + \text{H.c.} \right). \]

Upon bosonization, the corresponding operators read
\[ O_{\text{sp}}^{ij} = \cos \sqrt{\pi} (\Phi_i - \Phi_j) \cos \sqrt{\pi} (\Theta_i - \Theta_j), \]
\[ O_{\text{cdw}}^{ij} = \cos 4\pi (\Theta_i - \Theta_j), \]
\[ O_{\text{sc}}^{ij} = \cos 4\pi (\Phi_i - \Phi_j). \]

The dimensions of these operators at the SLL fixed point (4) are readily evaluated and are given by
\[ \eta_{\text{cdw}}^{(N)}(N) = 2 \int_{q_\perp} \frac{1 - \cos(Nq_\perp)}{K(q_\perp)}, \]
\[ \eta_{\text{sc}}^{(N)}(N) = 2 \int_{q_\perp} [1 - \cos(Nq_\perp)] K(q_\perp), \]
\[ \eta_{\text{sp}}^{(N)} = \frac{1}{4} (\eta_{\text{cdw}} + \eta_{\text{sc}}), \]
where \( N = |i - j| \). A necessary condition for this SLL phase to be stable is thus that
\[ \eta_{\text{cdw}}^{(N)}, \eta_{\text{sc}}^{(N)} > 2 \text{ and } \eta_{\text{cdw}}^{(N)} + \eta_{\text{sc}}^{(N)} > 8 \]
for all \( N \geq 1 \).
Stability of the SLL phase.—Before discussing the stability of SLL fixed points, let us first discuss the domain of validity of our RG approach. We are considering the scaling dimension of perturbing operators around the fixed points defined by the action (4). This action is defined provided the stiffness $K(q_{\perp})$ is positive everywhere in $[-\pi, \pi]$. Our study is thus restricted to the corresponding subspace of $K(q_{\perp})$. At the boundary of this subspace $K(q_{\perp})$ vanishes for some $q_{\perp}^{0} \in [0, \pi]$ [13]. The density correlations of the transverse wave vector $q_{\perp}^{0}$ then diverge, signaling an instability towards transverse charge-density wave ordering. In the transverse CDW, $\langle \rho_{\perp} \rangle \neq 0$, so then the total charge density on each chain is a function of the transverse position $i$. That the boundary corresponding to this transition plays a crucial role in the following analysis can be seen by inspecting the dimensions (7): $\eta_{\text{cdw}}^{(n)}$ can be significantly increased by the presence of the pole in the integrand [root of $K(q_{\perp})$] with $\eta_{\text{cdw}}^{(0)}$ being not much affected [9]. Hence it may be possible to have these operators irrelevant (i.e., with dimension greater than 2), for parameters in the vicinity of this boundary.

Stability analysis of a model $K(q_{\perp})$.—Here we consider the stability analysis for a concrete model of $K(q_{\perp})$. We thus look for a natural restriction to a finite number of terms of the Fourier expansion, which may allow a stable SLL fixed point. The simplest case turns out to be

$$K(q_{\perp}) = K_{0}[1 + \lambda_{1}\cos(q_{\perp}) + \lambda_{2}\cos(2q_{\perp})].$$

(9)

(As we discuss later, the model with just $\lambda_{1}$ included does not possess a stable SLL phase.) The requirement of positivity of $K(q_{\perp})$ restricts the range of $\lambda_{1}$ and $\lambda_{2}$, as shown in Fig. 1, to the region within the boundary $ABCD$ (henceforth simply denoted by $B$). The stability of the SLL to the perturbations (6) with $1 \leq (N = |i - j|) \leq 4$ is first determined [14]. The exponents in Eq. (7) are numerically evaluated and the results are shown in Fig. 1 which is a two-dimensional representation of the $(K_{0}, \lambda_{1}, \lambda_{2})$ space. The points marked are those for which there exists a range of $K_{0}$ values where the above operators are all irrelevant. The corresponding values of $K_{0}$ are all greater than 1 (repulsive interactions on the chains). All the stable fixed points are thus found close to the boundary $B$.

In an attempt to define stable RG fixed points, one may worry about more general perturbing operators. Indeed, inclusion of the operators (6) with $5 \leq N \leq 10$ does not significantly change the results. However, it turns out that general four point operators, such as simultaneous hopping of fermions from chain $i$ to $i + 1$ and from $j$ to $j + 1$ [15], have a dramatic effect on the stability region which is now much reduced. Thus, these operators are found to be relevant at a large number of the previously stable points. There are, however, some remaining fixed points, clustered close to the boundary $B$, which are shown as squares in Fig. 1.

Thus, as expected from the argument given above, all stable fixed points are found close to the boundary $B$. This can be physically understood as follows. Let us recall that anywhere on $B$ the system is on the verge of a transverse CDW instability. Clearly, if a transverse CDW is actually realized, it would frustrate crystallization of the fermions since the average spacing between the particles (i.e., the density) is now different on the different chains. Strong fluctuations of the transverse CDW kind occur as the boundary $B$ is approached. Such fluctuations prevent the locking-in of density modulations along the chain, hence defeating the crystal instability and stabilizing the SLL. This mechanism will be less effective if the wave vector $q_{\perp}^{0}$ is commensurate with the transverse lattice, i.e., if $q_{\perp}^{0} = 2\pi m/n$ for some integers $m$, $n$. Then, the longitudinal density waves on chains separated by $n$ can lock in and give rise to the crystal instability (provided $n$ is not too large). This idea is confirmed on inspecting the range of $K_{0}$ for which the SLL phases exist. In Fig. 2 the corresponding range of $K_{0}$ is plotted as a function of the transverse CDW instability period $2\pi/q_{\perp}^{0}$ (that it is nearest to). Drastic reduction of the SLL stability occurs at commensurate transverse wave vectors. This also allows us to understand the absence of stable SLL in the model with $\lambda_{2} = 0$. Then, the only existing transverse CDW has wave vector $\pi$ (for $\lambda_{1} = 1$) and hence for strong enough repulsive interactions, we expect the longitudinal density waves on next nearest neighbor chains ($N = 2$) to lock and lead to a crystalline instability. This turns out to be a correct expectation as, by including the $N = 2$ CDW operator, we do not find a stable SLL in this model.

The physical mechanism identified here allows us to generalize our results to models beyond the simple ones considered so far, including Luttinger liquids coupled in three dimensions. In general we expect the SLL to be

![FIG. 1. SLL fixed points for the model in (9). The allowed parameter range for which $K(q_{\perp})$ is positive is within the curve ACBD. For each point shown, there exists a finite range of $K_{0}$ for which the SLL is found to be stable against (a) CDW, SC, and SP perturbing operators of Eq. (5) with $N \leq 4$ shown by the small dots and (b) including general four point operators with $N \leq 10$ shown as squares.](image-url)
stabilized for renormalized couplings in the vicinity of an incommensurate transverse CDW transition. Thus the experimental search for such a phase in quasi-1D systems is most likely to be rewarded in the vicinity of an incommensurate transverse CDW state.

An approach very similar to the one taken in this paper can be applied to bosonic models at any filling, coupled anisotropic spin chains, or vortex lattices in anisotropic superconductors. These, together with the more complicated case of spinful fermions will be discussed in a forthcoming publication [6]. As disorder is known to modify significantly the behavior of a single Luttinger liquid, it may also be highly interesting to study its effect on the new phase described in this paper.

In conclusion, we have studied in detail the occurrence of the sliding Luttinger liquid phase in a simple three parameter model. We find that the region of stability of this phase is restricted to be close to the boundary where the harmonic boson theory breaks down and a soft mode appears, which signals a transverse CDW instability. Strong incommensurate transverse CDWs are particularly effective at frustrating crystal formation and enhancing the stability of the SLL. We expect these conclusions to apply also to more general models than the simple one considered in this paper.

During the completion of this work, we became aware of a similar study by Emery et al. [16]. However, the crucial role played by incommensurate transverse CDW fluctuations in stabilizing the SLL phase was not unveiled in [16], leading to different conclusions on the model with only nearest neighbor couplings ($\lambda_2 = 0$). In particular, we want to stress that we have found stable SLL fixed points only close to the transition towards transverse CDW phases.

It is a pleasure to thank T. Senthil for suggestions on this problem and continuous moral support, L. Balents, M. P. A. Fisher, K. Damle, A. Lopatin, T. Giamarchi, S. L. Sondhi, and F. D. M. Haldane for discussions, and I. Gruzberg for a careful reading of the manuscript. A. V. thanks the ITP, Santa Barbara for hospitality during part of this work. This work was supported by NSF Grants No. DMR-9528578 and No. PHY94-07194.

[4] See the recent review by F. von Oppen et al., cond-mat/0002087.
[8] Although the electrons in quantum Hall effect stripes are spin polarized, the physics in that case differs in many details from what we consider here, due to the presence of the soft translational mode and the magnetic field.
[11] More generally, one might consider a nonlocal potential $K_{ij}(|x - x'|)$. However, for smooth, short ranged potential, its Fourier transform may be expanded in a power series: $\tilde{K}_{ij}(k) = \tilde{K}_{ij}(0) + (k^2/2)\tilde{K}_{ij}''(0) + \cdots$, which leads to terms containing additional spatial derivatives, which are irrelevant at the SLL fixed points.
[12] Thus in the sliding Luttinger liquid, both the CDW phase field $\Theta_i$ and the Josephson phase field $\Phi_i$ can slide relative to each other at no cost.
[13] The case of double roots is considered in [6].
[14] Choosing the set of perturbing operators for a physical situation is an involved issue, which will depend on details of the system and the small but, nevertheless, finite temperature at which it is considered. Then, operators that have small bare values and are not further generated in the RG could be unimportant, even if relevant.
[15] More precisely, we consider operators of the form $\psi_{i\alpha}^{\dagger}\psi_{j\beta}^{\dagger}\psi_{k\gamma}\psi_{l\delta}$, where $\alpha = \pm 1$ for ($L/R$) and $\alpha + \beta = \gamma + \delta$ by momentum conservation. We have taken the maximum separation between any two fermion operators to be $\pm 10$. The ones that destabilize the SLL points usually involve simultaneous hops on pairs of nearest neighbor chains. These could well be present or generated in physical systems.