

Readings: Read set 3 of lecture notes; read tutorial by Ong et al. on Berry-phase theory of AHE. Read an introductory article on fractional quantum Hall effect (e.g., in volume edited by Prange and Girvin).

Credit: Students receiving 2 units should solve any 2 problems.

1. (a) Explain why the Berry curvature in a Bloch electron system vanishes everywhere in k -space in systems that have both inversion and time-reversal symmetry. What does this say about the anomalous Hall effect in such a system? (b) Construct a 2D band structure with two nondegenerate bands and Chern number 1 for the lower band.

2. Construct a 1D tight-binding model with two inequivalent sites per unit cell and a nonzero electrical polarization from the Berry-phase formula. Define a periodic parameter λ in your model that will pump a single electronic charge when changed from 0 to 2π .

3. Exercises on spin chains: (a) Take three spin-half sites in a triangle with equal antiferromagnetic exchange couplings, i.e.,

$$H = J \sum_{\langle ij \rangle} \mathbf{s}_i \cdot \mathbf{s}_j. \quad (1)$$

What is the ground state energy of this three-site Heisenberg model? What is the ground state of the XX model (i.e., leaving out the s^z coupling)? (b) Consider two sites with $s = 1$ spins. What is the ground state energy of the above Heisenberg interaction? Consider an additional biquadratic interaction $\beta J(\mathbf{s}_i \cdot \mathbf{s}_j)^2$. Compute the ground state energy of the two spins with this additional interaction as a function of β . Explain why such an interaction is redundant for $s = 1/2$.

4. Compute the normalized lowest Landau level eigenfunctions of a two-dimensional electron in a rotationally symmetric gauge $\mathbf{A} = (-By/2, Bx/2, 0)$. Check that the areal density is as expected: $n = \frac{1}{2\pi\ell^2}$, ℓ the magnetic length. Hint: exploit rotational symmetry and note that these eigenfunctions take an especially simple form in complex notation $z = x + iy$.